TIMED AUTOMATA

GOALS OF TODAY'S LECTURE

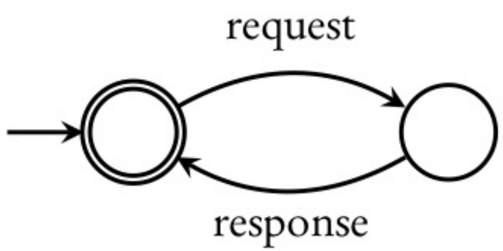
- 1. Motivations
- 2. Timed words & timed languages
- 3. Timed automaton model
- 4. Examples

Why do this course?

Automata (*Finite State Machines*) are **good abstractions** of many real systems

hardware circuits, communication protocols, biological processes, . . .

Automata can model many **properties** of systems



every request is followed by a response

System	Property
↓	↓
Automaton $\mathcal{A}$	Automaton $\mathcal{B}$

System → Automaton $\mathcal{A}$

Property → Automaton $\mathcal{B}$

Does system **satisfy** property?

System $\downarrow$ Automaton $\mathcal{A}$

Property $\downarrow$ Automaton $\mathcal{B}$

$$\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{B})?$$

Does system **satisfy** property?

Model-checking

System → Automaton $\mathcal{A}$

Property → Automaton $\mathcal{B}$

$$\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{B})?$$

Does system **satisfy** property?

In practice...

Huge system

Property

In practice...

Huge system	Property
↓	↓
Higher-level description	Higher-level description

In practice...

Huge system → Higher-level description → (translation) → Automaton $\mathcal{A}$

Property → Higher-level description → (translation) → Automaton $\mathcal{B}$

Model-Checker

$$\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{B})?$$

In practice...

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Model-Checker

$$\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{B})?$$

Some model-checkers: SMV, NuSMV, SPIN, . . .

In practice...

Huge system → Higher-level description → (translation) → Automaton $\mathcal{A}$

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Model-Checker

$$\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{B})?$$

Some model-checkers: SMV, NuSMV, SPIN, . . .

Turing Awards: Clarke, Emerson, Sifakis and Pnueli

Automata are **good abstractions** of many real systems

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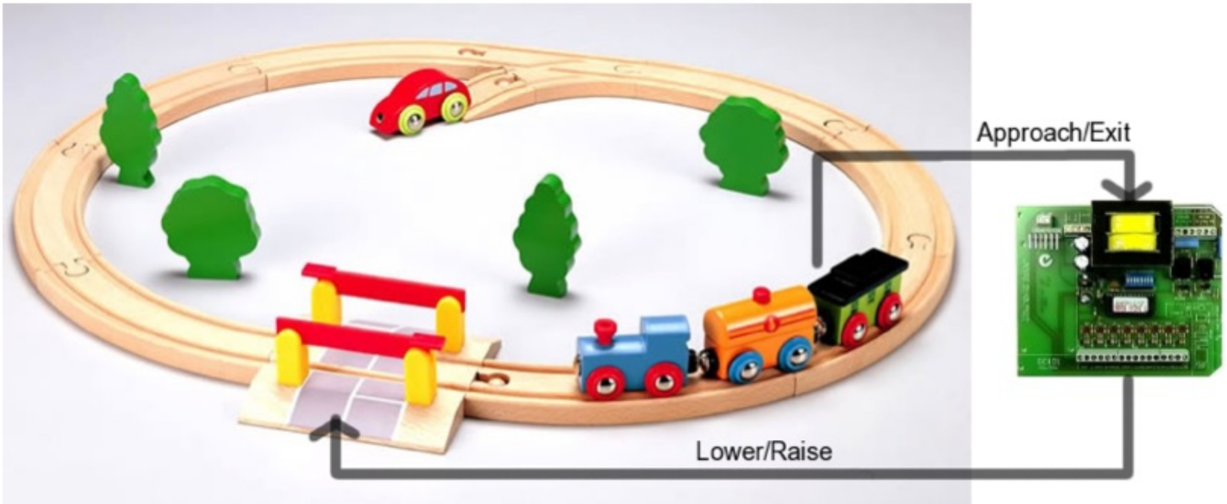
Our course: Automata for **real-time** systems



Picture credits: F. Herbreteau

pacemaker, vehicle control systems, air traffic controllers, . . .

Timed Automata

R. Alur and D. Dill in early 90s

Timed Automata

R. Alur and D. Dill in early 90s

Some model-checkers: UPPAAL, KRONOS, RED, . . .

TCHECKER

Goals of our course

- Understand **language theoretic** properties of timed automata
- Study **algorithms** used in model-checkers

Model-checking caters to **both theory** enthusiasts and **practice** enthusiasts

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this course is a good starting point for model-checking real-time systems

Timed languages and timed automata

Σ : alphabet $\{a, b\}$

Σ^* : words $\{\varepsilon, a, b, aa, ab, ba, bb, aab, \dots\}$

$L \subseteq \Sigma^*$: language $\longrightarrow$ *property over words*

$L_1 :=$ {set of words starting with an " a "}

$\{a, aa, ab, aaa, aab, \dots\}$

$L_2 :=$ {set of words with a non-zero even length }

$\{aa, bb, ab, ba, abab, aaaa, \dots\}$

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Finite automata, pushdown automata, Turing machines, ...

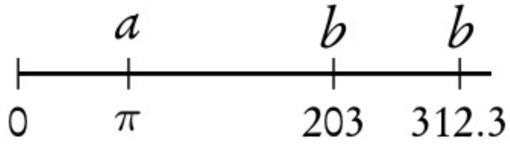
Σ : alphabet $\{a, b\}$

$T\Sigma^*$: timed words

$(aa;\ 0.8, 2.5)$ $(abb;\ \pi, 203, 312.3)$

Σ : alphabet $\{a, b\}$

$T\Sigma^*$: timed words



$(aa;\ 0.8, 2.5)$ $(abb;\ \pi, 203, 312.3)$

a b b,
1 2.2 2.2

$$(w, \tau)$$

Word — Time sequence

$w = a_1 \dots a_n$

$a_i \in \Sigma$

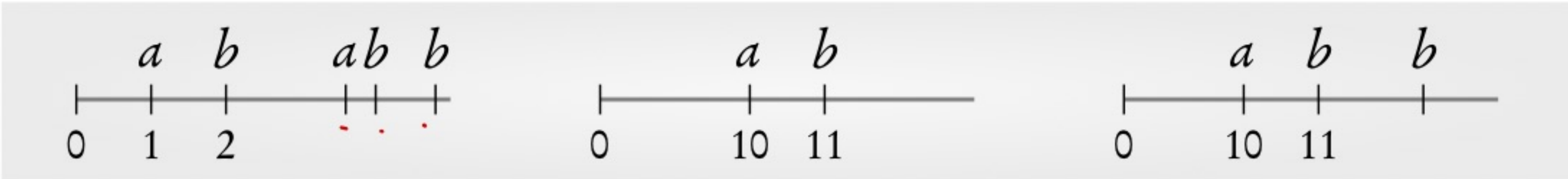
$\tau = \tau_1 \dots \tau_n$

$\tau_i \in \mathbb{R}_{\geq 0}$

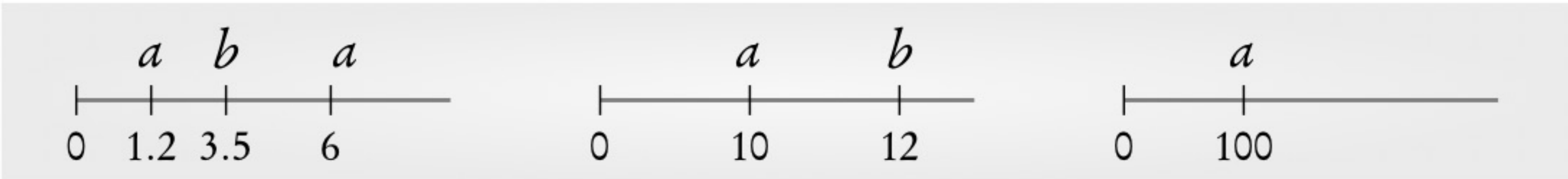
$\tau_1 \leq \cdots \leq \tau_n$

$L \subseteq T\Sigma^*$: Timed language $\longrightarrow$ *property over timed words*

$$L_1 := \{(\, ab(a+b)^*, \tau\,) \mid \tau_2 - \tau_1 = 1\}$$



$$L_2 := \{\, (w, \tau) \mid \tau_{i+1} - \tau_i \geq 2 \text{ for all } i < |w|\}$$



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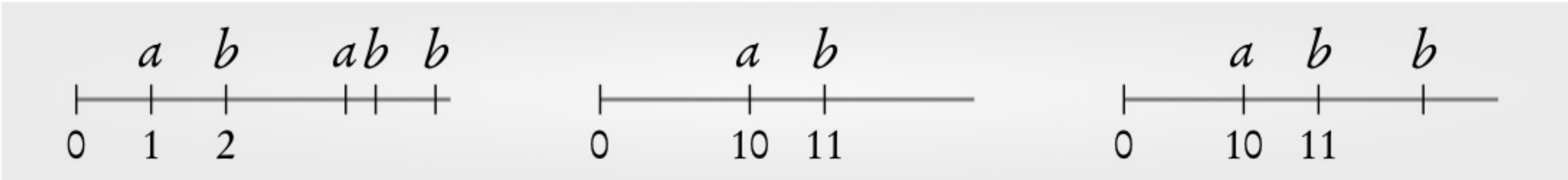

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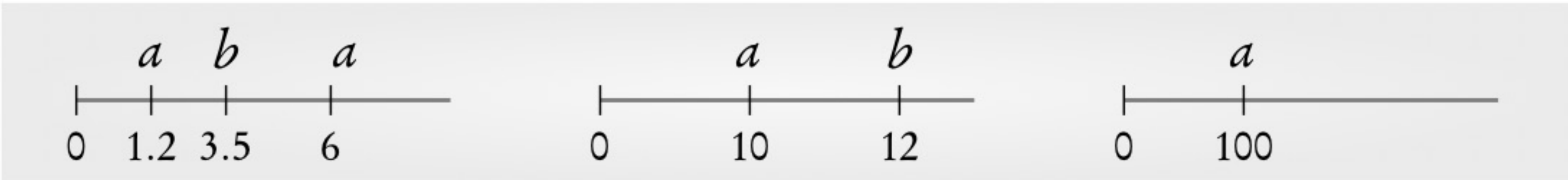
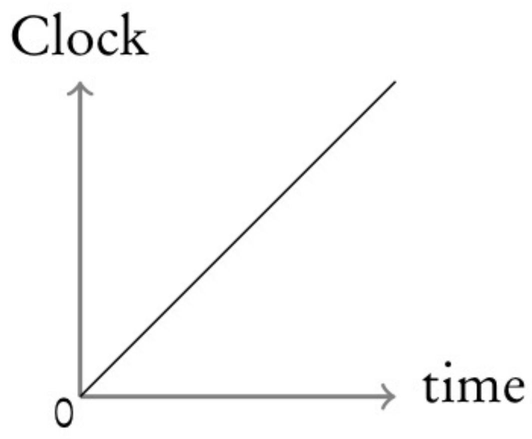


Timed automata

Timed automaton: Finite automaton + Finite no. of *Clocks*



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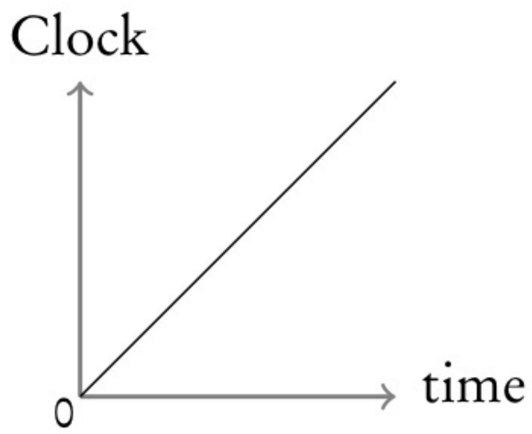
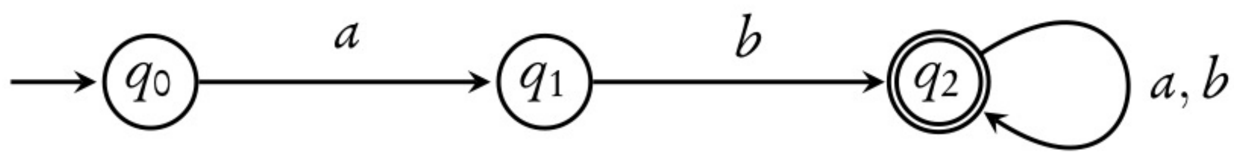


$$\{(\ ab(a+b)^*, \tau) \mid \tau_2 \leq 2\}$$



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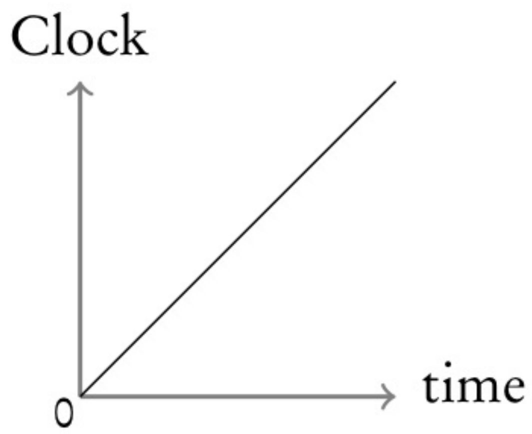

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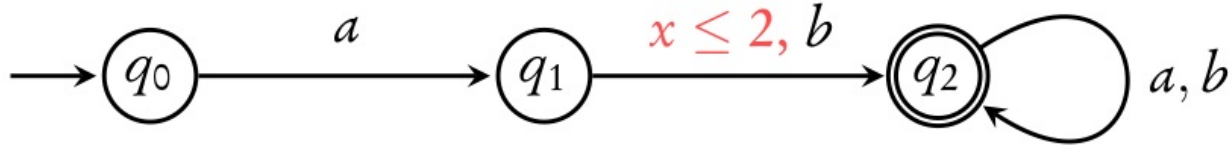

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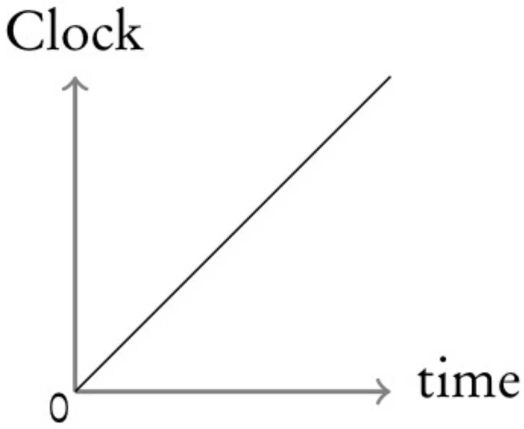

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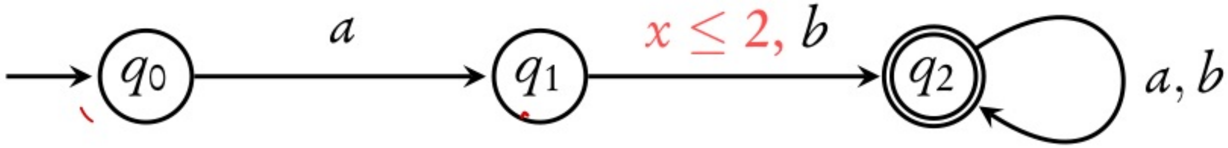

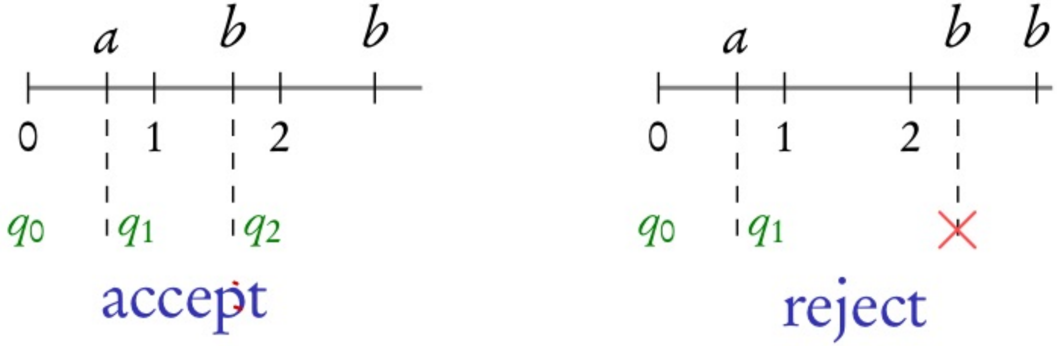

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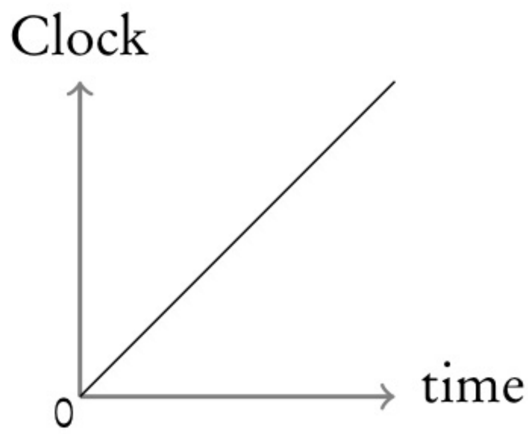
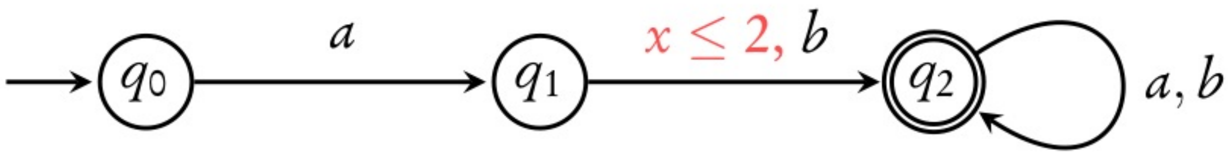


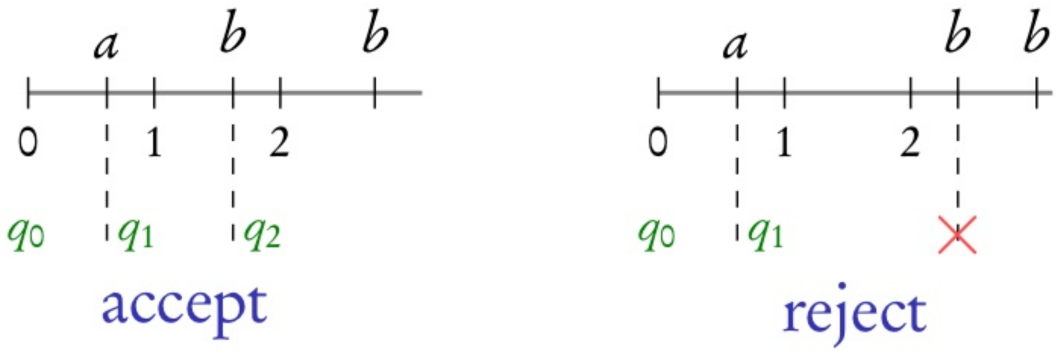
Guards

$$\phi := x \leq c \mid x \geq c \mid \neg\phi \mid \phi \wedge \phi$$

$$x \in Clocks\,,\ c \in \mathbb{Q}_{\geq 0}$$

$$\{(\, ab(a+b)^*, \tau) \mid \tau_2 \leq 2\}$$





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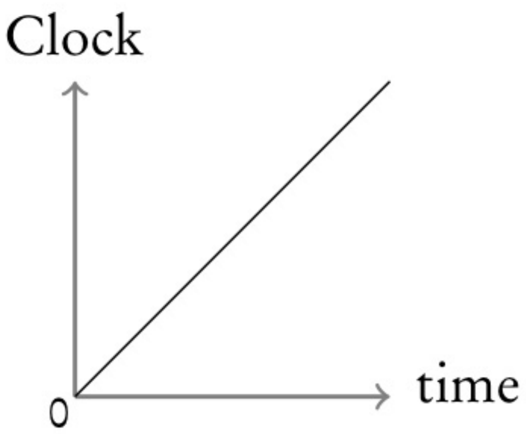

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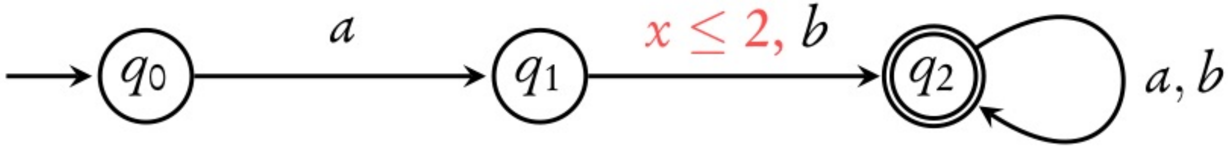

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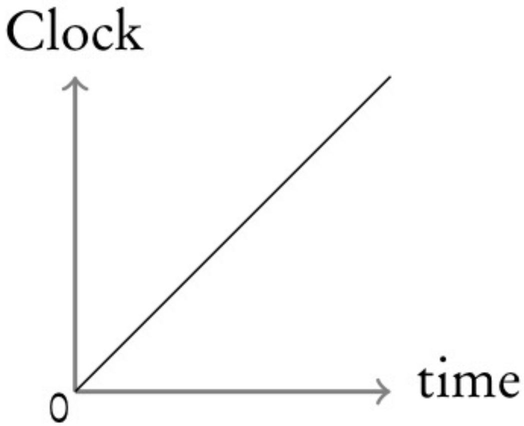

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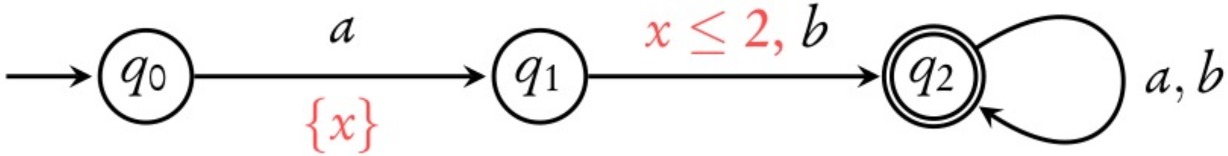
Resets

$$\{(\,ab(a+b)^*, \tau) \mid \tau_2 - \tau_1 \leq 2\}$$



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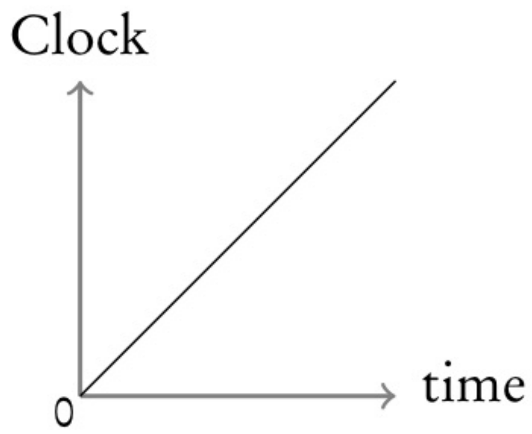

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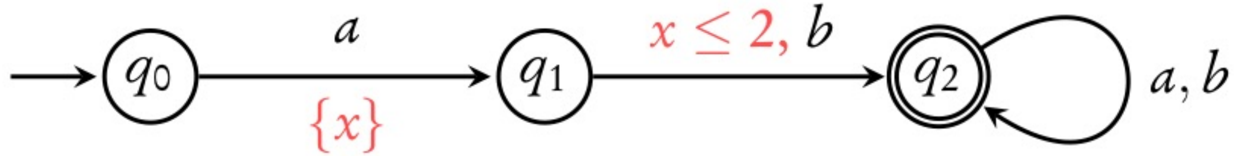
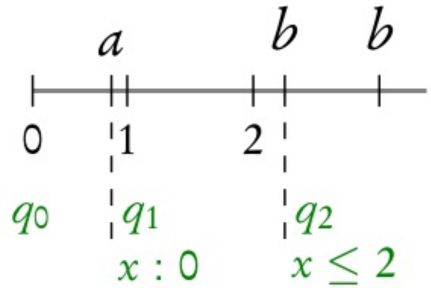
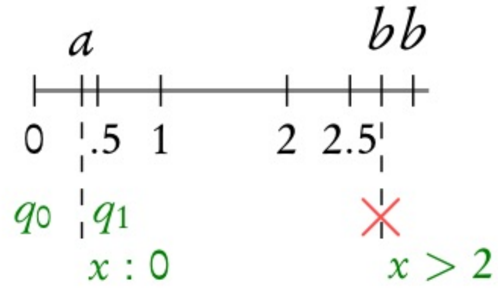




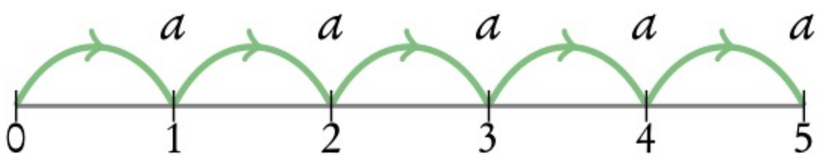
accept



reject

$$L_3 := \{ \, (\, a^k, \tau \,) \mid k > 0, \; \tau_i = i \; \text{ for all } i \leq k \}$$

An "a" occurs in every integer from $1, \ldots, k$

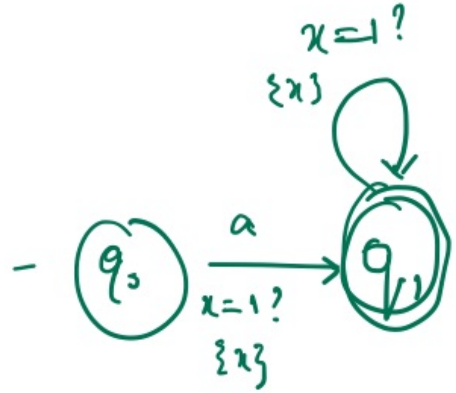


(a, 1) ✓

(a, 1.1) ✗

(aa, 1, 2) ✓

(aaaa, 1, 3, 5, 6) ✗



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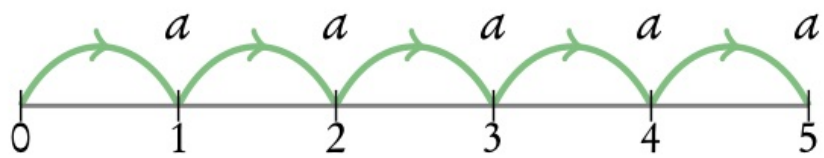
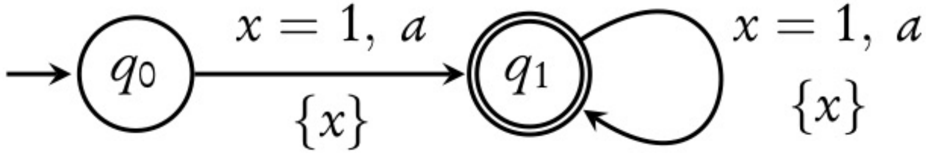
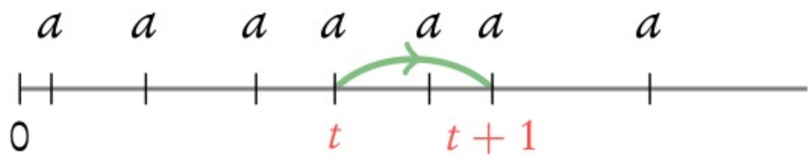




$$L_4 := \{ \, (\, a^k, \tau \,) \mid \text{exist } i, j \text{ s.t. } \tau_j - \tau_i = 1 \}$$

There are 2 "*a*"s which are at distance 1 apart



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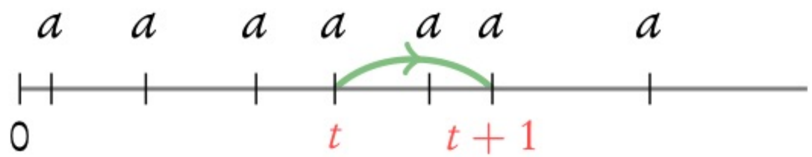
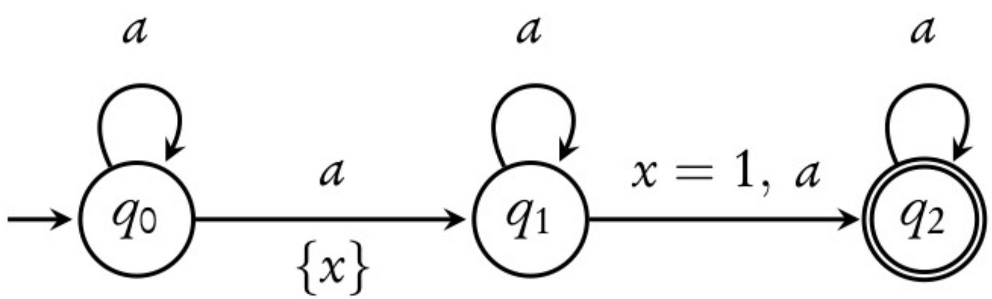
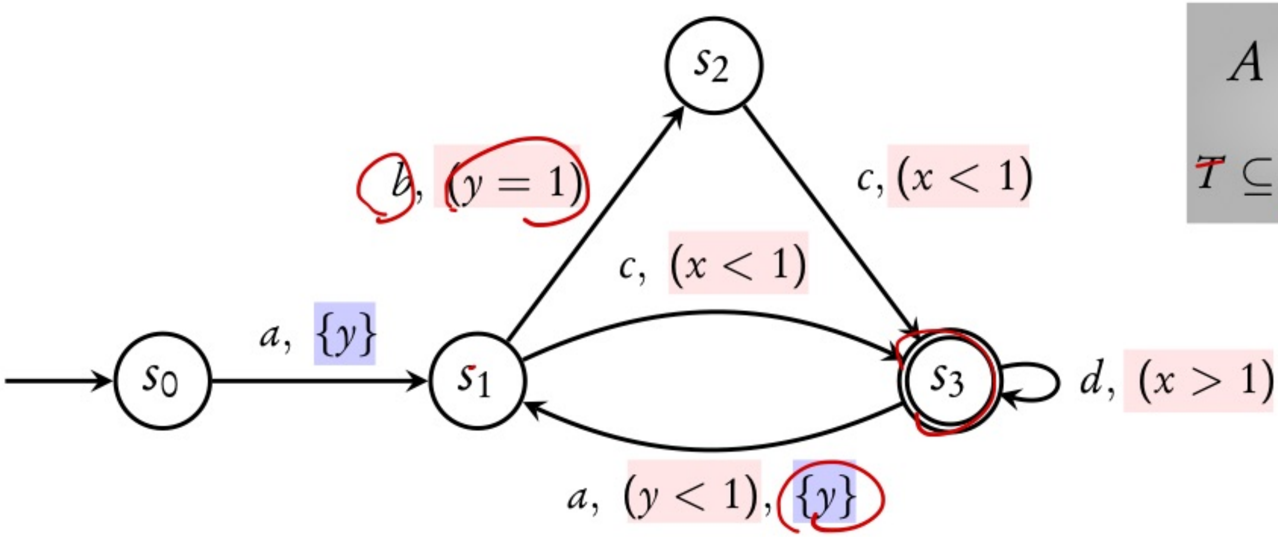






Clocks
Transitions
Initial
final

$$A = (Q, \Sigma, X, T, Q_0, F)$$

$$T \subseteq Q \times \Sigma \times \text{guard} \times \text{reset} \times Q$$

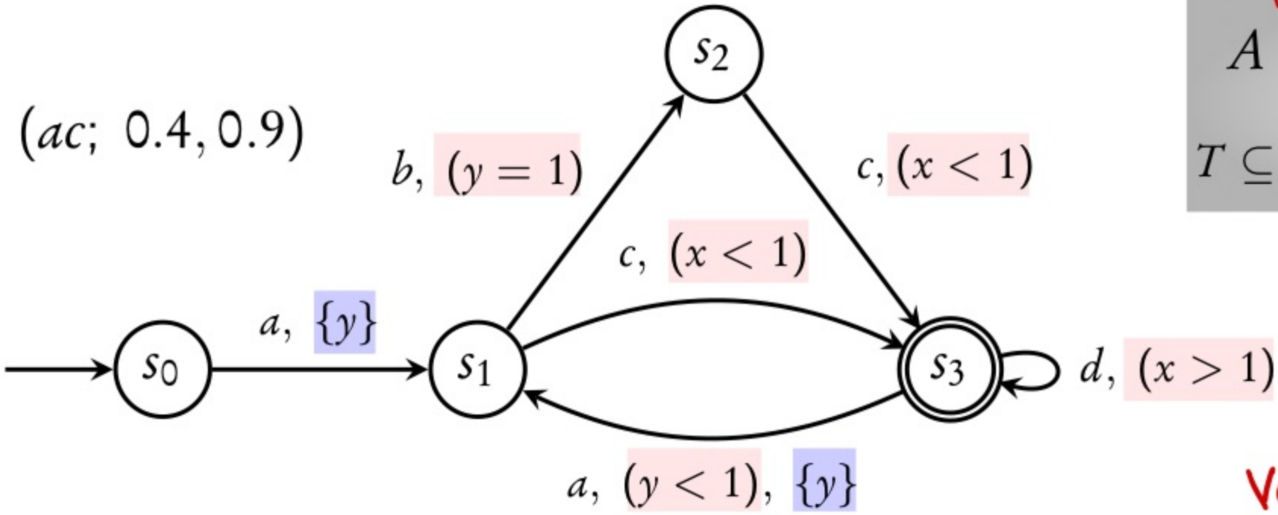
$\Phi(X)$ 2^X

Semantics is a timed language: set of timed words that are accepted by the automaton

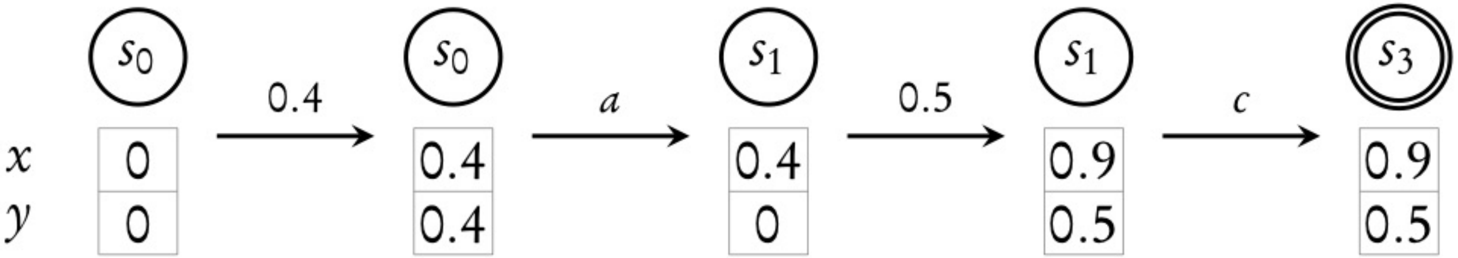
$$A = (Q, \Sigma, X, T, Q_0, F)$$

$$T \subseteq Q \times \Sigma \times \text{guard} \times \text{reset} \times Q$$

$(ac;\ 0.4, 0.9)$



Valuation: tuple of reals $\mathbb{R}_{\geq 0}^{|X|}$



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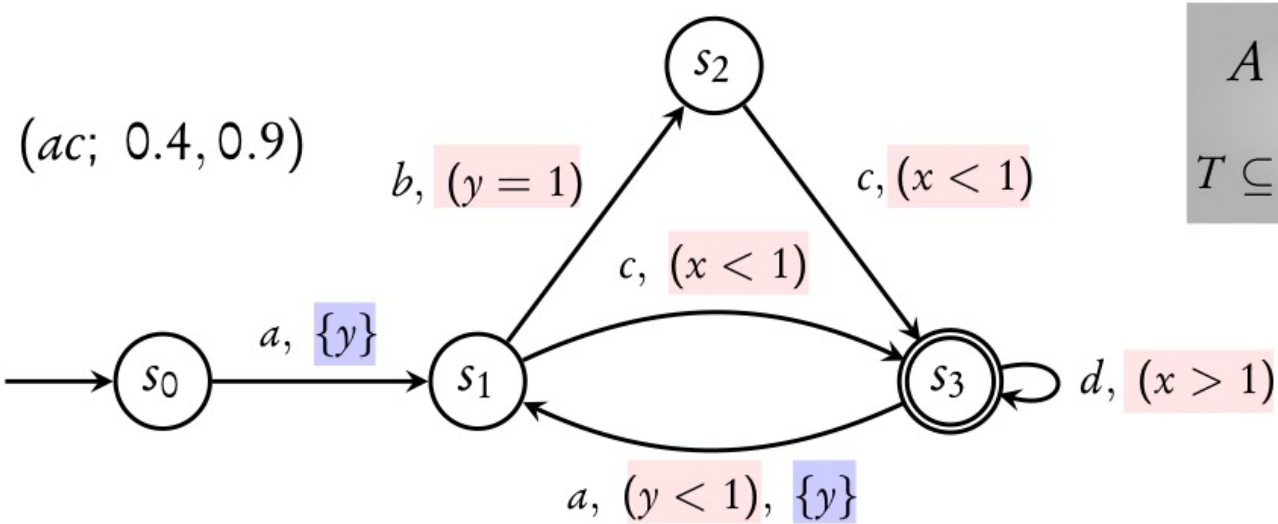

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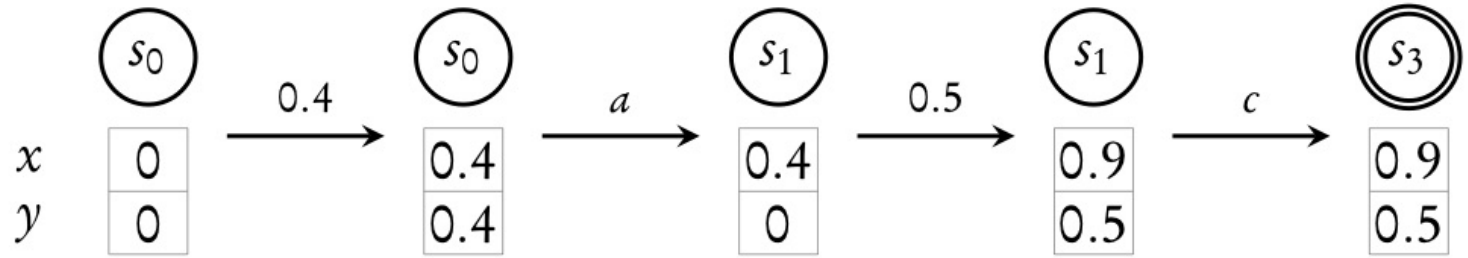


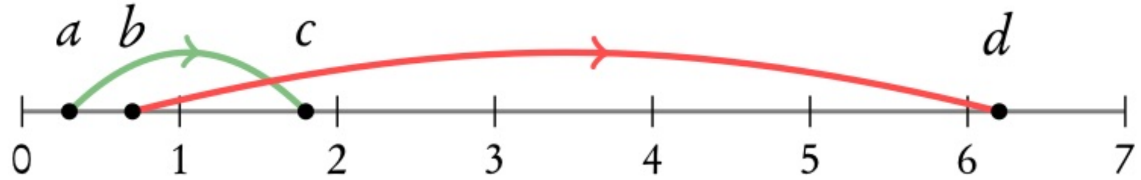
Run of A over $(a_1 a_2 \ldots a_k;\ \tau_1 \tau_2 \ldots \tau_k)$ $\qquad \delta_i := \tau_i - \tau_{i-1};\ \tau_0 := 0$

$$(q_0, v_0) \xrightarrow{\delta_1} (q_0, v_0 + \delta_1) \xrightarrow{a_1} (q_1, v_1) \xrightarrow{\delta_2} (q_1, v_1 + \delta_2) \ \cdots\ \xrightarrow{a_k} (q_k, v_k)$$

$(w, \tau) \in \mathcal{L}(A)$ if A has an **accepting** run over (w, τ)

$$L_5 := \{ \, (\, abcd.\Sigma^*, \tau \,) \mid \tau_3 - \tau_1 \leq 2 \text{ and } \tau_4 - \tau_2 \geq 5 \}$$

Interleaving distances



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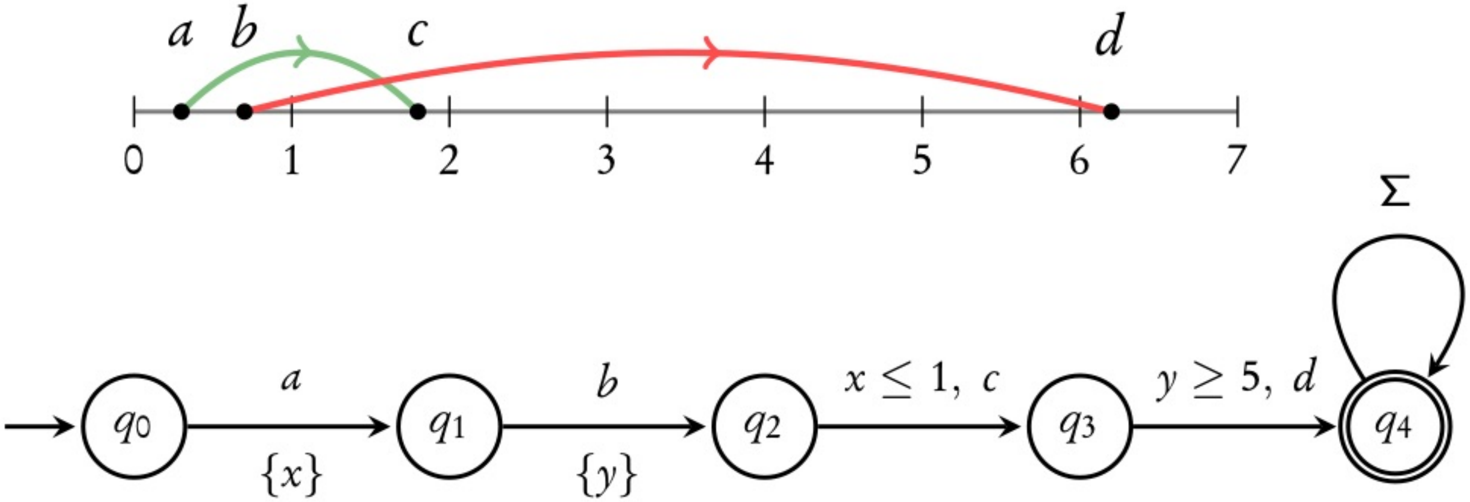
Interleaving distances



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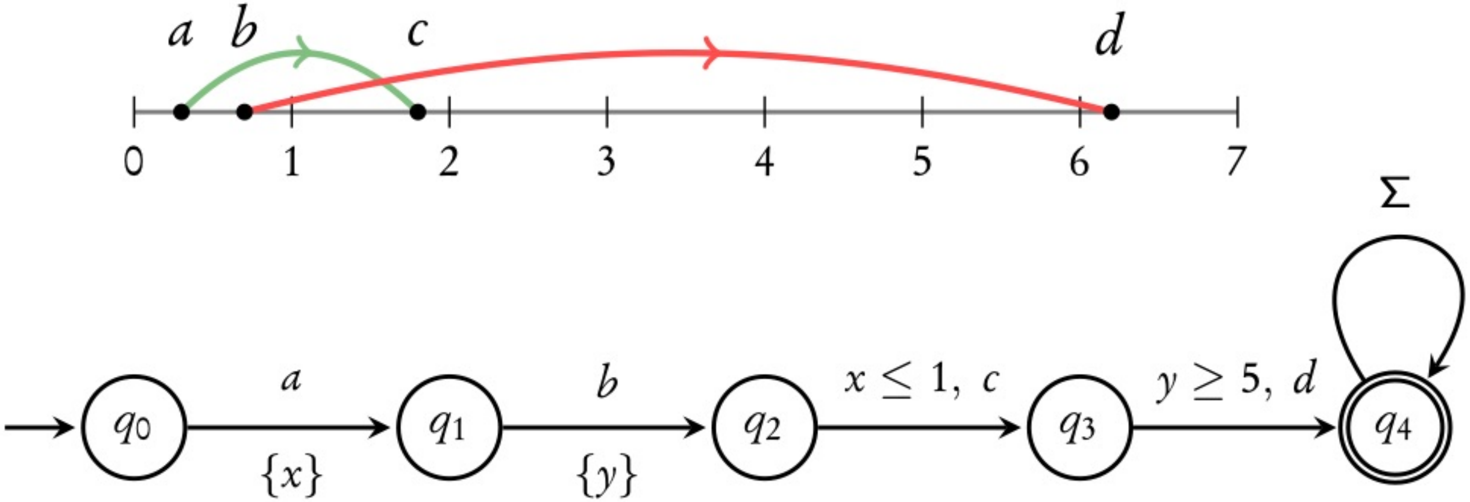
Interleaving distances



Exercise: Prove that L_5 cannot be accepted by a one-clock TA.

n interleavings $\Rightarrow$ need n clocks

$n + 1$ clocks more expressive than n clocks

$$\{\ (a^k, \tau) \mid \tau_{i+2} - \tau_i \leq 1 \text{ for all } i \leq k-2\}$$

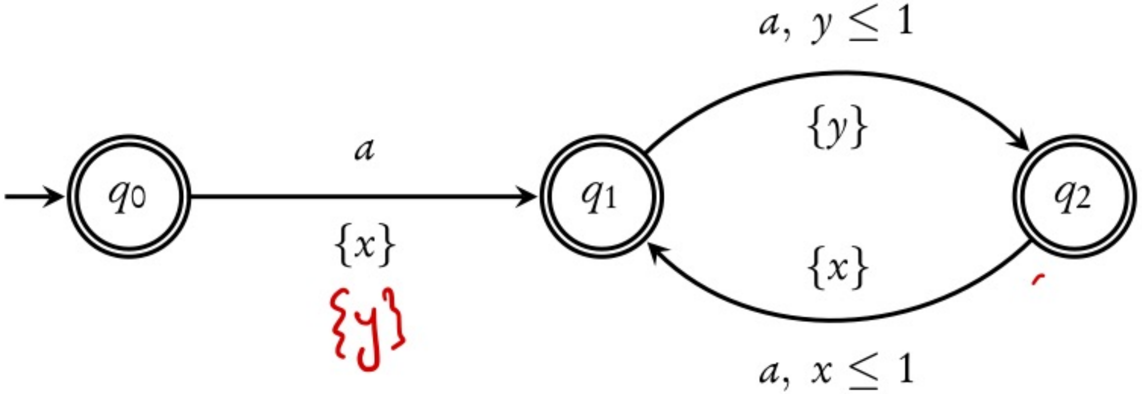
$$\{ (a^k, \tau) \mid \tau_{i+2} - \tau_i \le 1 \text{ for all } i \le k-2 \}$$

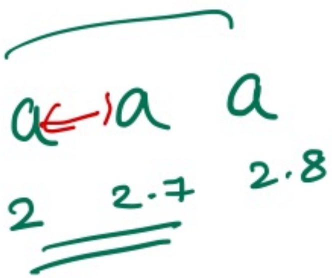
$\tau_0 = 0$

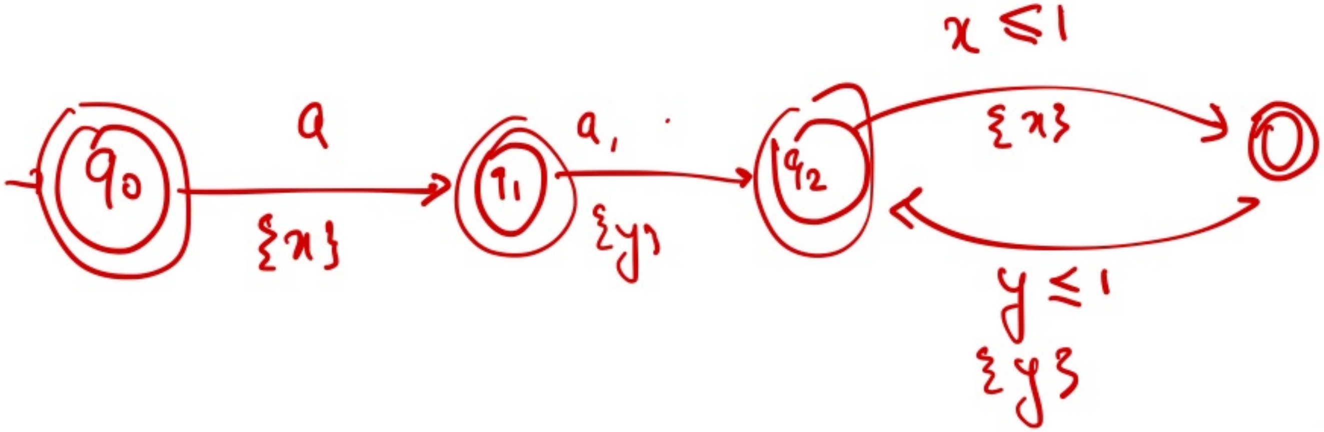
$0 \le i \le k-2$

$k \ge 0$

$\tau_2 - \tau_0 \le 1$



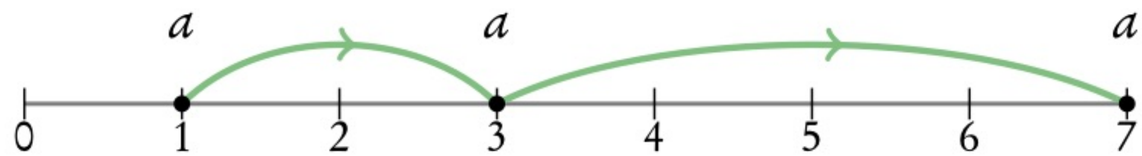




SEEN SO FAR...

- Runs of a timed automaton
- 1 clock < 2 clock < .....

$$L_6 := \{\ (\ a^k, \tau\ )\ |\ \tau_i \text{ is some integer for each } i\}$$



LECTURE 1 - Summary

- Timed automata, runs
- Expressive power increases with clocks
- Role of the max constant